COMMENTS ON THE ARTICLE [1]

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The aim of this note is to update the proof of [1, Proposition 3].

Firstly, we prove the following lemma.

Lemma 1. *Let C be a nonempty subset of a Banach space E, $\{T_n\}$ a sequence of mappings of C into E with a common fixed point, and $z \in C$ an asymptotic fixed point of $\{T_n\}$. Then there exist a* bounded *sequence $\{x_n\}$ in C and a subsequence $\{x_{n_i}\}$ of $\{x_n\}$ such that $x_n - T_n x_n \to 0$ and $x_{n_i} \rightharpoonup z$.*

Proof. By assumption, there exist a sequence $\{y_m\}$ in C and a subsequence $\{y_{m_i}\}$ of $\{y_m\}$ such that $y_m - T_m y_m \to 0$ and $y_{m_i} \rightharpoonup z$. Let u be a common fixed point of $\{T_n\}$ and define a sequence $\{x_n\}$ in C by

$$x_n = \begin{cases} y_{m_i} & \text{if there exists } i \in \mathbb{N} \text{ such that } n = m_i; \\ u & \text{if } n \neq m_i \text{ for all } i \in \mathbb{N}. \end{cases}$$

Then we can verify that $x_{m_i} \rightharpoonup z$ and $x_n - T_n x_n \to 0$. □

Corrected proof of [1, Proposition 3]. By assumption, it is clear that

$$\hat{\mathrm{F}}(\{S_n T_n\}) \supset \mathrm{F}(\{S_n T_n\}) \supset \mathrm{F}(\{S_n\}) \cap \mathrm{F}(\{T_n\}) = \hat{\mathrm{F}}(\{S_n\}) \cap \hat{\mathrm{F}}(\{T_n\}).$$

Thus it is enough to show that $\hat{\mathrm{F}}(\{S_n T_n\}) \subset \hat{\mathrm{F}}(\{S_n\}) \cap \hat{\mathrm{F}}(\{T_n\})$. Let $z \in \hat{\mathrm{F}}(\{S_n T_n\})$. Then $z \in D$ and it follows from Lemma 1 that there exist a *bounded* sequence $\{x_n\}$ in D and a subsequence $\{x_{n_i}\}$ of $\{x_n\}$ such that $x_n - S_n T_n x_n \to 0$ and $x_{n_i} \rightharpoonup z$. Thus [2, Lemma 3.6] implies that $x_n - T_n x_n \to 0$. Therefore $z \in \hat{\mathrm{F}}(\{T_n\})$. Moreover, [2, Lemma 3.6] also implies that $T_n x_n - S_n T_n x_n \to 0$. Since $\{T_n x_n\}$ is a sequence in C, $T_{n_i} x_{n_i} = (T_{n_i} x_{n_i} - x_{n_i}) + x_{n_i} \rightharpoonup z$, and C is closed and convex, it follows that $z \in C$ and hence $z \in \hat{\mathrm{F}}(\{S_n\})$. This completes the proof. □

References

[1] K. Aoyama, *Asymptotic fixed points of sequences of quasi-nonexpansive type mappings*, Banach and function spaces III (ISBFS 2009), 2011, pp. 343–350. MR3013135

[2] K. Aoyama, F. Kohsaka, and W. Takahashi, *Strongly relatively nonexpansive sequences in Banach spaces and applications*, J. Fixed Point Theory Appl. **5** (2009), no. 2, 201–224. MR2529497 (2010h:47076)

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